SOLUTION SET X

EXERCISE X.1 : CHANGE OF POWER

By convention, the Y index is used for the values of the wye assembly and the Δ index for the values of the delta assembly.

A. In a balanced three-phase system, the current in the neutral is zero. So nothing happens if we remove the link to the neutral.

Wye assembly

B. $U_{load,Y} = \frac{U_l}{\sqrt{3}} = \underline{231\,V}$

C. $Z_{load} = \sqrt{R^2 + \frac{1}{(\omega C)^2}} = \underline{19{,}9\,\Omega} \quad \Rightarrow \quad I_{load,Y} = \frac{U_{load,Y}}{Z_{ph}} = \underline{11{,}6\,A}$

The phase shift φ is equal to the argument of impedance: $\varphi = \operatorname{arctg}\left(\frac{-1/\omega C}{R}\right) \cong \underline{-59{,}84\,°}$

D. $P_{load,Y} = U_{load,Y} \cdot I_{load,Y} \cos\phi = \underline{1{,}347\,kW} \quad \Rightarrow \quad P_{tot,Y} = 3 \cdot P_{load,Y} = \underline{4{,}040\,kW}$

$Q_{load,Y} = U_{load,Y} \cdot I_{load,Y} \sin\phi = \underline{-2{,}317\,k\,\mathrm{var}} \quad \Rightarrow \quad Q_{tot,Y} = 3 \cdot Q_{ph,Y} = \underline{-6{,}951\,k\,\mathrm{var}}$

$Q < 0 \Rightarrow$ the reactive power is supplied to the network.

Delta assembly

B. $U_{load,\Delta} = U_l = \underline{400\,V}$

C. $I_{load,\Delta} = \frac{U_{load,\Delta}}{Z_{load}} = \underline{20{,}1\,A}$

The phase difference does not depend on the assembly type: $\varphi = \underline{-59{,}84\,°}$

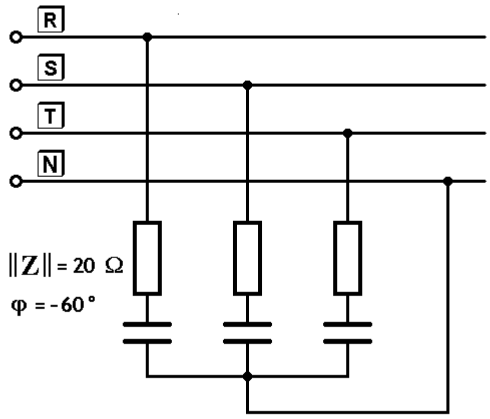
D. $P_{load,\Delta} = U_{load,\Delta} \cdot I_{load,\Delta} \cos\phi = \underline{4{,}040\,kW} \quad \Rightarrow \quad P_{tot,\Delta} = 3 \cdot P_{load,\Delta} = \underline{12{,}120\,kW}$

$Q_{load,Y} = U_{load,Y} \cdot I_{load,Y} \sin\phi = \underline{-6{,}951\,k\,\mathrm{var}} \quad \Rightarrow \quad Q_{tot,Y} = 3 \cdot Q_{load,Y} = \underline{-20{,}853\,k\,\mathrm{var}}$

Conclusion : By connecting the same impedances $\underline{Z}$ in a triangle on the same network, the powers are tripled.

EXERCISE X.2 : ADAPTATION TO TWO DIFFERENT NETWORKS

A.



B. In the delta configuration, the phase voltage is $U_{load,\Delta} = U_l = \underline{230\ V}$

In the wye configuration, the phase voltage is $U_{load,Y} = \frac{U_l}{\sqrt{3}} \cong \underline{230\ V}$

The same phase voltages on the same impedances give the same active and reactive powers. In both cases, the phase current is given by :

$$I_{load} = \frac{U_{load}}{|\underline{Z}|} = \underline{11{,}5\ A}$$

C. Delta assembly : $I_{l,\Delta} = \sqrt{3} \cdot I_{load} = \underline{19{,}91\ A}$

Wye assembly : $I_{l,Y} = I_{load} = \underline{11{,}5\ A}$

Conclusion : The powers remain the same; on the other hand, the currents in the lines change. To protect the user, it is necessary to reduce the size of the protective fuses when passing from the triangle to the star.

EXERCISE X.3 : EQUIVALENT IMPEDANCES

The equivalence of the delta and wye impedances is expressed by the two conditions :

$$I_{l,\Delta} = I_{l,Y} \qquad (1a)$$

$$I_{load,\Delta} \cdot U_{load,\Delta} \cos\phi_\Delta = I_{load,Y} \cdot U_{load,Y} \cos\phi_Y \qquad (1b)$$

Now we have the following relations, with the phase voltage U :

$$U_{load,\Delta} = \sqrt{3} \cdot U \qquad U_{load,Y} = U \qquad (2)$$

$$I_{load,\Delta} = \frac{U_{load,\Delta}}{Z_\Delta} = \frac{\sqrt{3} \cdot U}{Z_\Delta} \qquad I_{load,Y} = \frac{U_{load,Y}}{Z_Y} = \frac{U}{Z_Y} \qquad (3)$$

$$I_{l,\Delta} = \frac{3 \cdot U}{Z_\Delta} \qquad I_{l,Y} = \frac{U}{Z_Y} \qquad (4)$$

$$Z_\Delta = \sqrt{R_\Delta^2 + \left(\frac{1}{\omega C_\Delta}\right)^2} \qquad Z_Y = \sqrt{R_Y^2 + \left(\frac{1}{\omega C_Y}\right)^2} \tag{5}$$

$$\varphi_\Delta = -\mathrm{arctg}\left(\frac{1}{\omega R_\Delta C_\Delta}\right) \qquad \varphi_Y = -\mathrm{arctg}\left(\frac{1}{\omega R_Y C_Y}\right) \tag{6}$$

The conditions (1a) and (1b) of equivalence of the impedances become :

$$\frac{3U}{Z_\Delta} = \frac{U}{Z_Y} \tag{7a}$$

$$\frac{3\,U^2}{Z_\Delta}\cdot\cos\varphi_\Delta = \frac{U^2}{Z_Y}\cdot\cos\varphi_Y \tag{7b}$$

Substituting (7a) in (7b), we obtain :

$$\cos\varphi_\Delta = \cos\varphi_Y \qquad \Rightarrow \qquad R_\Delta\cdot C_\Delta = R_Y\cdot C_Y \tag{8}$$

Moreover, equation (7a) can be written :

$$Z_Y = \frac{Z_\Delta}{3} \tag{9}$$

$$R_Y^2 + \left(\frac{1}{\omega C_Y}\right)^2 = \frac{1}{3^2}\left[R_\Delta^2 + \left(\frac{1}{\omega C_\Delta}\right)^2\right] \tag{10}$$

$$\frac{\omega^2 R_Y^2 C_Y^2 + 1}{\omega^2 C_Y^2} = \frac{1}{3^2}\cdot\frac{\omega^2 R_\Delta^2 C_\Delta^2 + 1}{\omega^2 C_\Delta^2} \tag{11}$$

By making use of the relation (8) in (11):

$$C_Y = 3\cdot C_\Delta = \underline{540\ \mu F}$$

Finally, by the relationship (8) :

$$R_Y = \frac{1}{3}\cdot R_\Delta = \underline{7\ \Omega}$$